FINITE ELEMENT MODELING, RECONCILIATION AND EVALUATION OF PREDICTIVE ACCURACY: A CASE STUDY

John R. Red-Horse* and Kenneth F. Alvin†

Structural Dynamics and Vibration Control Dept.
Sandia National Laboratories, Albuquerque, NM

Abstract

Results of a case study in high precision finite element model updating are presented. The goal of the study was to investigate the effects that model form errors have on the overall test/analysis model reconciliation process by tracking the characteristics of an updated model through a structural modification. Two classes of analytical model were considered for each configuration. Modal tests were performed on each structural configuration and the results of these tests were subsequently used as criteria for updating selected physical parameters in both finite element models and assessing the predictive accuracy of the models. The updates were performed using Bayes Estimation. The results demonstrate that a high degree of accuracy can be achieved for both classes of analytical model through systematic reconciliation with test-estimated modal parameters, and that much of this accuracy is maintained through the design modification.

1. Introduction

Finite element modeling has become, at the close of the 20th century, a necessary tool in the design of nearly all civil and aerospace structural systems. This acceptance has been predicated on a number of factors; among them, the development of robust and efficient methods for developing and checking models via advanced visualization. Unfortunately, the time and resource constraints placed on the most skilled of analysts invariably forces a number of modeling assumptions to be made—assumptions which may have major impacts on the predictive accuracy of the model.

To a certain extent, the accuracy of a given model and the validity of the assumptions which underlie its development are often taken for granted. In fact, absolute accuracy is often not even an issue. For example, in design, models can be utilized safely for comparison studies where the qualitative effects of design alternatives are under investigation; and, in civil structural applications, large margins of safety can mitigate the need for precise predictive model-based calculations. Finally, in many applications specific to structural dynamics the issue of model accuracy is addressed by performing modal tests where the goal is to establish the suitability of a given structure for a given task.

But the current trend is toward a greater reliance on analytical modeling for many of these applications. The rationale is that employing computational models can result in savings, in both time and cost, relative to design procedures that rely more heavily on building prototypes and performing extensive tests on them. However, this shift is not without some risks and one issue that simply must be addressed is that of evaluating the absolute accuracy of models and, correspondingly, establishing the validity of the associated modeling assumptions.

In general, it is clear that model range and application should be taken into consideration when determining when a given model is acceptable for a given application. Bridge models most certainly do not require the same level of accuracy as, say, space-based precision pointing structures.

Most update procedures, by their very nature, will tend to improve a model's performance when evaluated relative to a given specific criteria set. The more interesting exercise is to inspect its behavior outside this comparison set, where model form error is likely to affect accuracy. This is the case when models are inspected outside the frequency range of interest or when they are used to make predictions of response prop-

* Senior Member Technical Staff, Structural Dynamics and Vibration Control Dept. (Mail Stop 0439), Sandia National Laboratories, P.O. Box 5800, Albuquerque, NM 87185

† Senior Member Technical Staff, Structural Dynamics and Vibration Control Dept., Member AIAA

erties after a structural modification has taken place.

In this study, we investigate these issues by conducting a case study on a simple structure which was analyzed and tested both in its original, baseline, state and in a modified configuration. The baseline structure consisted of a welded frame composed of thin-wall, rectangular steel tubing formed into a symmetric two rung ladder. This baseline was then modified by removing portions of each leg in an antisymmetric fashion. These structures display a surprising degree of complexity due, among other things, to welded joint compliance, indeterminate load paths and plate-like behavior exhibited by the walls of the tubes.

2. Baseline Structure

A modal test was first performed for the baseline structure. The instrumentation setup, which is shown in Figure 1, consisted of six accelerometers at each cross-section in an attempt to associate both translational and rotational degrees of freedom (DOF) with the nodes of the corresponding beam. Excitation was effected through the use of an impact hammer and frequency response function (FRF) data were obtained using standard data reduction techniques. A modal model was then obtained using Polyreference [1] as implemented in SDRC's TDAS software package. A photograph of the baseline structure in its test configuration is shown in Figure 2.

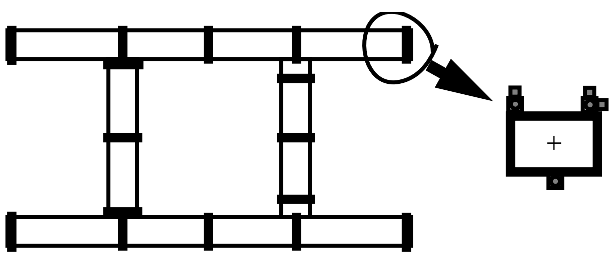

Figure 1: Instrument Setup for Baseline Structure

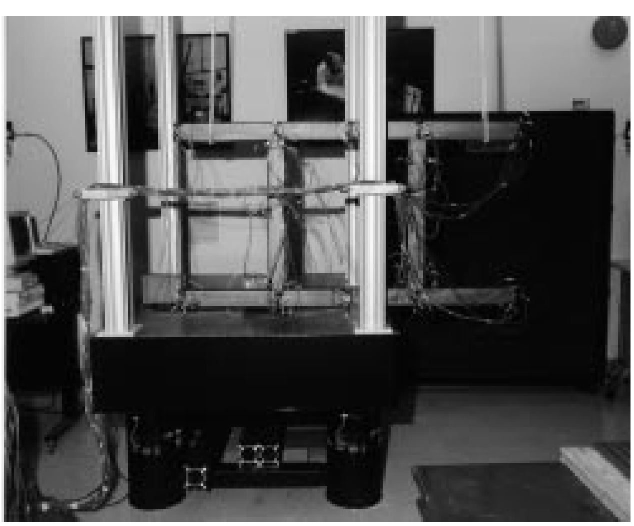

Figure 2: Modal Test Configuration of Baseline Design

2.1 Beam Element Model

A MSC/NASTRAN beam finite element model was developed that consisted of CBEAM elements with six DOF springs representing the welds. GRID locations were established to coincide with the physical locations of the accelerometers and these GRIDs were attached to the beams via multi-point constraints. The use of CBEAM elements was dictated by the need to correctly model the torsional inertia of the cross-section. The accelerometer masses were also incorporated into the model. A plot of the beam model geometry is shown in Figure 3.

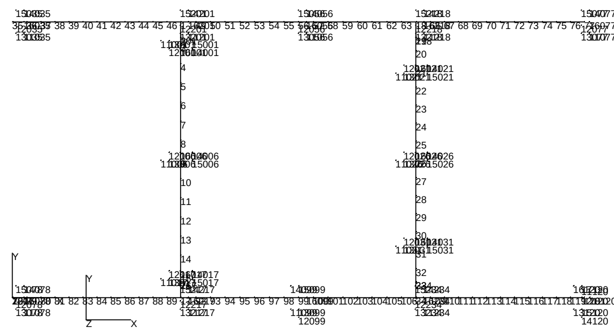

Figure 3: Beam Finite Element Model of Baseline Design

Table 1 contains data specific to the initial model. The information includes a frequency comparison as well as a listed modal assurance criteria (MAC) coefficient [2] for each pair of corresponding modes in the update bandwidth of $0 - 700$ Hz. A preliminary sensitivity analysis was performed which yielded a selection of physical parameters in the model that were suitable for updating. These parameters were the six spring coefficients at the joints (the coefficients were assumed to be common to each of the 4 joints) and the common beam cross-sectional properties: I_1, I_2, and J.

The model was updated with Bayes Estimation [3] as it exists in SSID [4] using the first 14 modal frequencies of the test-derived model in the observation vector (i.e. as criteria for reconciliation). Table 2 shows the results of this update procedure. The updated model exhibits marked improvement over the initial model as can be confirmed by close examination of the table. In Table 3, the estimated parameter movements and the final coefficients of variation (COV) for each parameter are listed. (This latter property is a measure of how much sensitivity that each parameter possesses for the elements in the chosen observation set).

Some points of note:

- The final model has modes which are correctly ordered relative to those in the test-derived modal model and for which the maximum frequency error is 3.7%. Compare this to the poor correspondence in the initial model, particularly for test mode 12, and frequency errors which were as large as 44%.

Table 1: Initial Beam Model/Test Correlation Results for Baseline Structure

Test Mode	Test Freq (Hz)	Model Mode	Model Freq (Hz)	%diff Freq	MAC
1	78.97	7	72.52	-8.16	0.9975
2	170.63	9	175.80	3.03	0.9965
3	174.47	8	162.87	-6.65	0.9930
4	214.72	10	207.67	-3.28	0.9983
5	250.91	11	256.49	2.22	0.9960
6	312.17	13	324.06	3.81	0.9710
7	315.79	12	315.48	-0.10	0.9553
8	317.77	15	372.82	17.33	0.9504
9	330.27	14	340.59	3.13	0.9968
10	432.52	16	460.00	6.53	0.9939
11	518.60	17	543.01	4.71	0.9890
12	563.65	20	815.51	44.68	0.8301
13	612.81	18	643.30	4.97	0.9828
14	674.36	19	694.94	3.05	0.8130

Table 2: Updated Beam Model/Test Correlation Results for Baseline Structure

Test Mode	Test Freq. (Hz)	Model Mode	Model Freq. (Hz)	%diff Freq.	MAC
1	78.97	7	78.80	-0.21	0.9978
2	170.63	8	169.67	-0.56	0.9966
3	174.47	9	174.67	0.11	0.9927
4	214.72	10.	218.27	1.65	0.9985
5	250.91	11	249.03	-0.75	0.9964
6	312.17	12	307.99	-1.34	0.9914
7	315.79	13	315.60	-0.06	0.9782
8	317.77	14	323.00	1.65	0.8724
9	330.27	15	324.11	-1.86	0.9548
10	432.52	16	435.32	0.65	0.9958
11	518.60	17	514.96	-0.70	0.9890
12	563.65	18	542.82	-3.70	0.8722
13	612.81	19	615.07	0.37	0.9740
14	674.36	20	673.28	-0.16	0.8351

- Generally, the MAC's improved. However, the MACs for test modes 8 and 9 dropped significantly. This can be attributed, in part, to the modal density in the 310 – 320 Hz frequency range and the fact that the difference vectors between those test modes and their corresponding analysis modes have high MACs with the closely neighboring test modes.

Table 3: Parameter Results from Baseline Structure Beam Model Update

Parameter	Final Value (relative to initial)	Initial COV	Updated COV
K_{uy}	0.4250	100 %	0.49 %
$K_{\theta x}$	0.2580	100 %	0.00153 %
$K_{\theta y}$	104.0	100 %	3.58 %
$K_{\theta z}$	1.4621	100 %	1.39 %
I_1	0.9415	3 %	0.00663 %
I_2	0.9178	3 %	0.0191 %
J	1.0091	3 %	0.00661 %

- The MAC's for test modes 12 and 14 continue to be low.
- Unfortunately, the modal properties for those modes that were outside the update frequency bandwidth still exhibited large errors.

2.2 Plate Element Model

To investigate the issue of model suitability, a plate model was developed using CQUAD4 elements. CBEAM elements were used to join the plates at the weld locations and at any location where plates joined one another at 90 degree angles. Figure 4 illustrates the fidelity achieved by the plate finite element model.

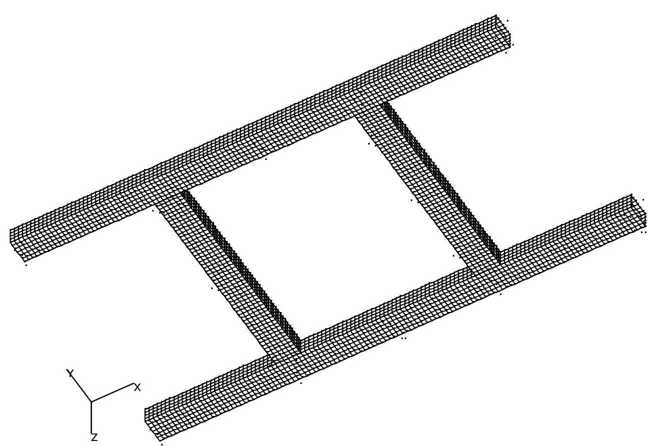

Figure 4: Plate Finite Element Model of Baseline Design

Tables 4 and 5 list the pre- and post-update model assessments, respectively, and similarly to Table 3, Table 6 contains parameter specific information on the updated parameters. Not surprisingly, perhaps, the initial model dem-

Table 4: Initial Plate Model/Test Correlation Results for Baseline Structure

Test Mode	Test Freq. (Hz)	Model Mode	Model Freq. (Hz)	%diff. Frequency	MAC
1	78.9674	1	77.702	-1.60%	0.998
2	170.6259	3	174.214	2.10%	0.997
3	174.4670	2	169.938	-2.60%	0.991
4	214.7231	4	217.851	1.46%	0.999
5	250.9062	5	258.566	3.05%	0.994
6	312.1717	8	322.748	3.39%	0.877
7	315.7890	7	321.063	1.67%	0.865
8	317.7661	6	310.536	-2.28%	0.978
9	330.2652	9	333.508	0.98%	0.993
10	432.5194	10	438.403	1.36%	0.995
11	518.5953	12	535.055	3.17%	0.988
12	563.6540	11	526.246	-6.64%	0.957
13	612.8141	13	612.955	0.02%	0.923
14	674.3648	14	678.556	0.62%	0.798

Table 5: Final Plate Model/Test Correlation Results for Baseline Structure

Test Mode	Test Freq. (Hz)	Model Mode	Model Freq. (Hz)	%diff Frequency	MAC
1	78.9674	1	77.51	-1.84%	0.998
2	170.6259	2	170.63	-0.03%	0.997
3	174.4670	3	174.47	0.58%	0.991
4	214.7231	4	213.57	-0.54%	0.999
5	250.9062	5	255.00	1.63%	0.994
6	312.1717	7	317.33	1.65%	0.993
7	315.7890	8	321.65	1.86%	0.982
8	317.7661	6	316.33	-0.45%	0.982
9	330.2652	9	326.91	-1.01%	0.995
10	432.5194	10	432.20	-0.07%	0.997
11	518.5953	11	525.15	1.26%	0.988
12	563.6540	12	566.76	0.55%	0.963
13	612.8141	13	602.72	-1.65%	0.958
14	674.3648	14	673.02	-0.20%	0.809

onstrates that the plate model has a trend towards better modal fidelity initially, but that certain problems, such as with test modes 6, 7 and 8, persist. In fact, our experiences with the beam model and in preliminary updating runs for the plate model led us to preclude the use of test modes 6, 7, and 8 in the observations vector. The updated model yields frequencies that are all within 2% error and, with the exception of mode 14, have high MAC values as well.

Table 6: Parameter Results from Baseline Structure Plate Model Update

Parameter	Final Value (relative to initial)	Initial COV	Updated COV
E_{global}	0.9515	1 %	0.021 %
E_{joint1}	0.9900	1 %	0.98 %
E_{joint2}	0.9866	1 %	0.99 %
E_{bar}	1.0303	5 %	5.00 %
$Spring$	20.00	2000 %	95. %
I_{plate}	1.3385	50 %	0.47 %

One significant advantage that was achieved by transitioning to the plate model was to capitalize on its ability to account for transverse plate bending that seems to be present in the physical model. Since the underlying assumptions of the beam formulation assume that the beam cross sections distort solely in the out-of-plane direction, these bending properties could not be captured in the beam model. Such an effect is difficult to observe in modal tests carried out with discretely located accelerometers and is motivation for investigating alternative methods possessing higher density measurement capacities, such laser-based acquisition techniques.

The modeling of a beam-like structure with plate elements to capture this behavior is not without complication, however. Although the plate element model accurately reflects the geometry of the structure in a highly detailed sense, the initial transverse bending stiffness of the plate elements was apparently too low. The effect of this error is that the initial plate model exhibited a much higher degree of cross-section warping and non-beam-like behavior than the actual structure. This parameter error accounted for the largest frequency and model shape discrepancies in the initial model, and an increase of 34% in the bending stiffness was required to achieve the desired correlation. Therefore, in a sense, the beam model, which has no in-plane warping, and the initial plate model, which exhibited a high degree of warping, bracket the behavior of the actual structure. This behavior has been most accurately captured, we feel, by the updated plate element model incorporating the increased bending stiffness parameter.

3. Modified Structure

One substantial test of model form is the robustness of the parameter values to actual model changes. Our approach was to modify the structure in such a way as to be considered non-trivial while avoiding anything complex enough to complicate the modeling process. The decision was made to remove portions of each leg of the baseline structure from opposite ends. This change, shown in Figure 5, rendered frequency and vector changes sufficient to satisfy the first objectives and added no new structural elements, such as joints, which might violate the latter. Table 7 summaries these effects in terms of the difference between the frequencies of the baseline structure and those of the modified structure, as predicted by the updated beam element model. The mode shape correlations also indicate substantial differences between the mode shapes of the two designs.

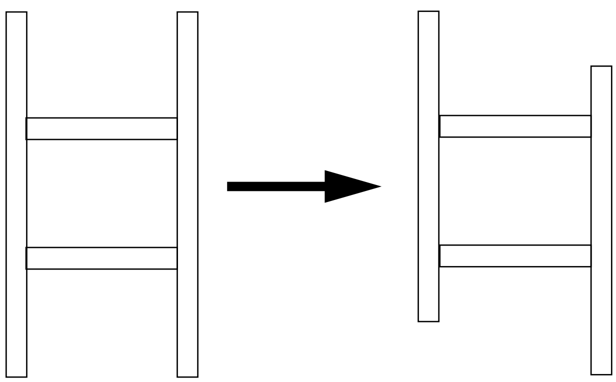

Figure 5: Modification of Ladder Structure

Tables 8 and 9 summarize the comparison of the beam

Table 7: Effects of Design Modification on Modal Parameters as Predicted by Beam Model

Nom. Design Mode #	Nom. Model Freq. (Hz)	Modified Design Mode #	Modified Model Frequency (Hz)	%diff Frequency	MAC
1	78.80	1	94.57	20.0%	0.945
2	169.67	3	245.53	44.7%	0.694
3	174.67	2	188.51	7.9%	0.987
4	218.27	4	266.25	22.0%	0.752
5	249.03	7	339.37	36.3%	0.616
6	307.99	5	312.07	1.3%	0.886
7	315.60	5	312.07	-1.1%	0.903
8	323.00	6	331.24	2.6%	0.909
9	324.11	4	266.25	-17.9%	0.701
10	435.32	3	245.53	-43.6%	0.779
11	514.96	7	339.37	-34.1%	0.806
12	542.82	9	581.36	7.1%	0.982
13	615.07	12	716.85	16.5%	0.665
14	673.28	10	628.94	-6.6%	0.723

Table 8: Correlation of Test with Initial Beam Model

Test Mode	Test-Identified Frequency (Hz)	Model Mode	Model Freq. (Hz)	%diff. Frequency	MAC
1	96.15	1	88.78	-7.66	0.9988
2	191.78	2	177.71	-7.33	0.9989
3	251.40	3	257.49	2.42	0.9987
4	269.94	4	262.97	-2.58	0.9977
5	321.83	5	319.68	-0.67	0.9941
6	327.33	7	380.85	16.35	0.9781
7	349.48	6	353.13	1.04	0.9943
8	497.66	8	502.88	1.05	0.9921
9	596.01	13	873.77	46.60	0.9237
10	634.15	10	676.08	6.61	0.9856
11	640.81	9	657.80	2.65	0.9754
12	714.26	11	750.63	5.09	0.9905
13	745.58	12	755.48	1.33	0.9923
14	853.90	16	1093.75	28.09	0.8456
15	943.05	14	948.63	0.59	0.2345
16	947.81	14	948.63	0.09	0.9068
17	968.91	15	955.22	-1.41	0.9424
18	1041.39	22	1426.17	36.95	0.5052
19	1106.86	17	1146.02	3.54	0.5517
20	1171.64	21	1399.64	19.46	0.6079

model of the modified structure using the initial and final parameter values achieved via the baseline beam model update. As expected, the frequencies in the initial model contain extremely large discrepancies (mode 9 is almost 50% in error) and the mode ordering is poor. Substituting the final parameter values into the model results in remarkably good modal correlation within the frequency band of interest (the first 14 modes) without further updating. The model form assumptions do, however, begin to surface as can be seen by inspecting the MAC column for both the in-band data and all of the out of band information.

Tables 10 and 11 show similar results for the plate models. Clearly, the plate model serves as a better approximation to the physical process. Using pre-update parameter values yields predictions which correlate well with the test data: frequencies in the 2-4% range error (with a couple of outliers), adequate modal ordering, and acceptable MACs. The post-update parameters values generally result in model improvements. Perhaps most surprising is the quality of the updated model outside of the frequency band of interest for which the original configuration had been reconciled: modal correspondence, frequency differences and MACs have all effectively

Table 9: Correlation of Test with Updated Beam Model

Test Mode	Test-Identified Frequency (Hz)	Model Mode	Model Frequency (Hz)	%diff. Freq.	MAC
1	96.15	1	95.22	-0.96	0.9992
2	191.78	2	189.33	-1.28	0.9990
3	251.40	3	247.74	-1.46	0.9990
4	269.94	4	268.44	-0.55	0.9984
5	321.83	5	312.94	-2.76	0.9949
6	327.33	6	332.69	1.64	0.9682
7	349.48	7	340.23	-2.65	0.9930
8	497.66	8	487.29	-2.08	0.9925
9	596.01	9	585.13	-1.83	0.8960
10	634.15	11	642.67	1.34	0.9776
11	640.81	10	629.69	-1.74	0.9815
12	714.26	12	722.54	1.16	0.9929
13	745.58	13	728.89	-2.24	0.9958
14	853.90	14	819.59	-4.02	0.7469
15	943.05	17	1152.57	22.22	0.5091
16	947.81	15	932.49	-1.62	0.8133
17	968.91	16	932.63	-3.74	0.7542
18	1041.39	21	1302.27	25.05	0.4552
19	1106.86	18	1189.15	7.43	0.8585
20	1171.64	20	1294.92	10.52	0.8072

Table 10: Correlation of Test with Initial Plate Model

Test Mode	Test-Identified Frequency (Hz)	Model Mode	Model Frequency (Hz)	%diff. Freq.	MAC
1	96.15	1	93.98	-2.25	0.9996
2	191.78	2	184.97	-3.55	0.9991
3	251.40	3	252.66	0.50	0.9992
4	269.94	4	270.18	0.09	0.9995
5	321.83	6	322.47	0.20	0.9948
6	327.33	5	317.77	-2.92	0.9892
7	349.48	7	352.84	0.96	0.9952
8	497.66	8	492.67	-1.00	0.9944
9	596.01	9	553.07	-7.21	0.9734
10	634.15	10	633.34	-0.13	0.9965
11	640.81	11	641.44	0.10	0.9954
12	714.26	12	711.83	-0.34	0.9954
13	745.58	13	742.52	-0.41	0.9978
14	853.90	14	806.26	-5.58	0.9806
15	943.05	15	888.95	-5.74	0.7113
16	947.81	16	938.31	-1.00	0.9104
17	968.91	17	950.03	-1.95	0.9619
18	1041.39	18	971.78	-6.68	0.8975
19	1106.86	19	1094.95	-1.08	0.9299
20	1171.64	22	1191.06	1.66	0.8583

converged.

4. Concluding Remarks

In this paper, the finite element modeling, reconciliation, and evaluation of predictive accuracy for the GM Flat Engine Cradle Simulant have been reviewed. Both beam element and plate element models were formulated and updated to the modal parameters estimated from a test of the baseline flat ladder. It was found that the beam element model required primarily an estimation of unknown joint compliance, together with small adjustments to the basic cross-sectional properties. The plate element model, on the other hand, required almost no adjustment of the joint compliance, but instead required adjustment of the basic transverse bending stiffness of the plate elements to eliminate a high degree of in-plane warping. Both updated models of the baseline structure were highly accurate in terms of their frequency and mode shape predictions, with the plate element model exhibiting the smallest errors.

The predictive accuracy of the updated models was evaluated through a design modification made to the baseline ladder structure. Two of the long rails were shortened in an antisymmetric manner in order to cause a significant change to the modal properties. The modified structure was tested and modal parameters estimated. The same design modifications were incorporated into the updated analytical models and those models were used to predict the modal parameters estimated by test. The results indicate that, although both models produced reasonably accurate predictions of the change in modal parameters, the plate element model was clearly superior. Furthermore, the accuracy of the modes above the spectrum used for the baseline design model reconciliation were studied. Again, the plate element model exhibits a much highly level of accuracy than that achieved by the beam element model.

Generally, model range and application should be taken into consideration when determining whether or not a given model is suitable for a given application. Most update procedures, by their very nature, will tend to improve a model's performance when evaluated relative to a given criteria set. Often the more interesting exercise is to inspect its behavior outside this comparison set as is the case when models are inspected outside the frequency range of interest or when they are used to make predictions of properties after a structural

Table 11: Correlation of Test with Updated Plate Model

Test Mode	Test Frequency (Hz)	Model Mode	Model Frequency (Hz)	%diff. Freq.	MAC
1	96.148	1	93.75	-2.50	0.9996
2	191.777	2	190.47	-0.68	0.9991
3	251.401	3	248.10	-1.31	0.9992
4	269.941	4	264.93	-1.86	0.9995
5	321.825	5	320.49	-0.41	0.9950
6	327.334	6	323.55	-1.16	0.9898
7	349.480	7	347.04	-0.70	0.9953
8	497.656	8	485.18	-2.51	0.9975
9	596.011	9	595.25	-0.13	0.9923
10	634.151	10	626.35	-1.23	0.9972
11	640.806	11	632.39	-1.31	0.9969
12	714.259	12	700.92	-1.87	0.9956
13	745.581	13	736.36	-1.24	0.9981
14	853.904	14	839.51	-1.69	0.9747
15	943.05	15	929.85	-1.40	0.7046
16	947.81	16	941.10	-0.71	0.9100
17	968.91	17	955.88	-1.35	0.9617
18	1041.39	18	1025.90	-1.49	0.9342
19	1106.86	19	1101.42	-0.49	0.9368
20	1171.64	20	1183.45	1.01	0.8729

modification has taken place. For the current case study, comparisons using both of these methods indicate that model form error does, in fact, affect a given model's accuracy. When models are under development, appropriate steps should be taken to address this issue.

REFERENCES

1. H. Vold and G. F. Rocklin. The numerical implementation of a multi-input modal estimation method for mini-computers. In Proceedings of the 1st IMAC, pages 542-548, Nov 1982.

2. D. J. Ewins. Modal Testing: Theory and Practice. John Wiley and Sons, New York, 1984.

3. A. H. Jazwinski. Stochastic Processes and Filtering Theory. Academic Press, New York, 1970.

4. T. K. Hasselman and J. D. Chrostowski. SSID-A computer code for structural system identification. Technical Report TR-91-1134-1, Engineering Mechanics Associates, Inc., 3820 Del Amo Blvd., Suite 318, Torrance, CA, Mar 1991.